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Technical Note

# Dimensional analysis for Gorter–Mellink counter flow convection in pressurized superfluid helium

Thomas C. Chuang *

*P&M Research Center, Vanung University, No. 1, Van Nung Road, Jung-Li, Taiwan, ROC*

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**Abstract**

Dimensional analysis of the heat transport through ducts filled with saturated He II is extended to pressurized conditions up to 20 bar. Simplified models are also presented for the ease of first order estimation of the GM-transport heat flux density, the temperature gradient and the limiting heat flux densities by relating calculations to reduced thermo physical properties, with respect to the properties at the lambda point. The data available show good support for the dimensionless GM-equation in pressurized He II within data scatter and thermo physical property uncertainty.


*Keywords:* Superfluid; Superconductor; Helium II; Low temperature; Heat transfer

## 1. Introduction

Superfluid liquid He II ($He^4$) is considered as a very unique and efficient coolant for high performance superconducting systems [1,2]. Much work has been done to understand the heat transport properties of He II where a large body of knowledge of its behavior at low temperatures and pressures are available. However, there are only a few comprehensive data sets available in suitable form in a particular application area. Therefore, the purpose of the present work is to present a simple set of functions which permit a quick selection of various system options. Empirical formulae are employed to relate the thermo physical properties to the reduced properties relative to the properties at lambda points. This leads to surprisingly simple functions which ease the calculation of the heat flux density at a given temperature and pressure. Experimental data are compared favorable with the dimensional analysis.

## 2. Laminar transport in zero net mass flow ($\Delta T \sim 0$)

At very low temperature difference, between the temperature of the heated end and the temperature of bath, the normal fluid flow in tubes is described by the Hagen-Poiseuille equations. One can generalize the laminar transport of He II with the following equation

$$N_q = N_{\nabla T} \tag{1}$$

where $N_q = \rho v_n L_c/\eta_n = qL_c/\eta_n ST$ and $N_{\nabla T} = \rho \nabla P_T L_c^3/\eta_n^2 = \rho^2 S \nabla T L_c^3/\eta_n^2$. Although the characteristics of the He II laminar flow are rarely evaluated in practical applications, its analysis helps to establish the dimensionless numbers which will be used as a basic framework for the analyses in the following sections.

* Tel.: +886 3 4515811x279; fax: +886 3 4513786.
*E-mail address:* tomchuang@msa.vnu.edu.tw

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**Nomenclature**

| | |
|---|---|
| $A_{GM}$ | Gorter–Mellink constant (turbulent reciprocal viscosity, m s/kg |
| $D$ | diameter, m |
| $f(T_\lambda)$ | a function related to thermodynamic properties at the lambda point |
| $g(T)$ | a function related to thermodynamic properties of He II |
| $K_{GM}$ | Universal Gorter–Mellink constant |
| $L$ | length, m |
| $L_c$ | characteristic length, m |
| $N_q$ | dimensionless number related to the heat flux density |
| $N_{\mathrm{grad}\,T}$ | dimensionless number associated with the temperature gradient |
| $P$ | pressure, bar |
| $q$ | heat flux density, W/cm$^2$ |
| $q_L$ | limiting heat flux density, W/cm$^2$ |
| $S$ | entropy, J/kg K |
| $S_\lambda$ | entropy at the lambda point, J/kg K |
| $T$ | temperature, K |
| $T_\lambda$ | temperature at the lambda point, K |
| $y(T_\lambda)$ | a function related to thermodynamic properties at the lambda point |
| $z(t_b)$ | a function related to thermodynamic properties of He II |
| $\rho$ | density, kg/m$^3$ |
| $\rho_n$ | normal fluid density, kg/m$^3$ |
| $\rho_s$ | superfluid density, kg/m$^3$ |
| $\eta_n$ | normal fluid shear viscosity, kg/m s |
| $\eta_\lambda$ | shear viscosity at the lambda point, kg/m s |

## 3. Transport at small $\Delta T$-values ($\Delta T \ll T$)

The counter flow of GM-transport in an insulated duct is usually written as

$$q = \rho_s S T \left\{ \frac{S|\nabla T|}{A_{GM}\eta_n} \right\}^{1/3} \tag{2}$$

($S$ = entropy per unit mass, $\eta_n$ is the shear viscosity of the normal fluid). There are two asymptotes for $A_{GM}$, one at low temperatures, the other one at high temperatures. Soloski and Frederking [3] demonstrated that at low temperatures, the macroscopic continuum approaches led to $(\eta_n A_{GM})^{-1} = \text{constant} = K_{GM}^3$, where $K_{GM}$ is of the order $10(\pi)^{1/3}$. At higher temperatures, near the lambda temperature ($T_\lambda$) $A_{GM}$ approaches the function $(\rho/\rho_s)$. For both constraints to be satisfied, the GM-transport property becomes

$$A_{GM} = K_{GM}^{-3}(\rho/\rho_s)/\eta_n \tag{3}$$

Insertion of Eq. (3) into (2) gives

$$q = K_{GM}\rho_s S T \left\{ \left(\frac{\rho_s}{\rho_n}\right)\left(\frac{\eta_n}{\rho}\right) S \mid \nabla T \mid \right\}^{1/3} \tag{4}$$

For the ease of estimating the heat flux density and the temperature gradient for a given GM duct design, one can use the reduced thermo physical properties for calculation. The reduced properties are evaluated with respect to the properties at the lambda point. The relationship between the reduced shear viscosity and the reduced temperature for various pressures is shown in Fig. 1 [4–7]. It is noted that the appearance and the shape of the reduce viscosity is substantially similar at different pressures. Eq. (4) can be further simplified so that an effective thermal conductance can be derived against reduced thermal physical properties.

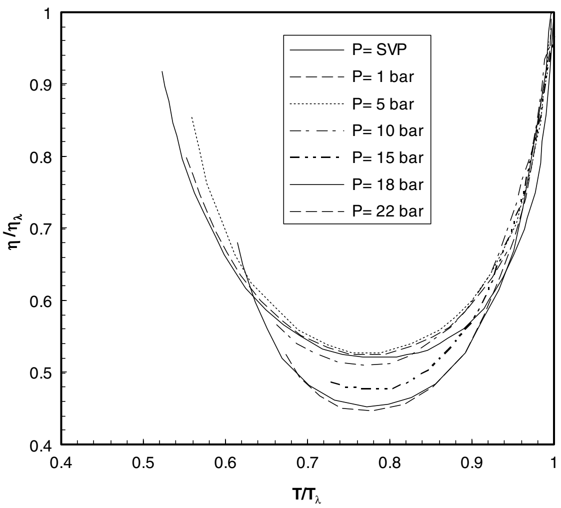


Fig. 1. The reduced viscosity versus the reduced temperature of He II at different pressures.

$$\frac{q^3}{\nabla T} = K_{GM}^3 \rho_\lambda^2 S_\lambda^4 T_\lambda^3 \eta_\lambda \times \left\{ \left(1 - \frac{\rho_n}{\rho}\right)^4 \left(\frac{\rho_n}{\rho}\right)^{-1} \left(\frac{\rho}{\rho_\lambda}\right)^2 \left(\frac{S}{S_\lambda}\right)^4 \left(\frac{T}{T_\lambda}\right)^3 \left(\frac{\eta}{\eta_\lambda}\right) \right\} \tag{5}$$

Apparently, it is easier to evaluate the lambda point properties and the dimensionless values separately. The following two functions are defined for the ease of calculation.

$$f(T_\lambda) = \left(\rho_\lambda^2 S_\lambda^4 T_\lambda^3 \eta_\lambda\right) \tag{6}$$

$$g(T) = \left(1 - \frac{\rho_n}{\rho}\right)^4 \left(\frac{\rho_n}{\rho}\right)^{-1} \left(\frac{\rho}{\rho_\lambda}\right)^2 \left(\frac{S}{S_\lambda}\right)^4 \left(\frac{T}{T_\lambda}\right)^3 \left(\frac{\eta}{\eta_\lambda}\right) \tag{7}$$

The effective thermal conductance can then be expressed as

$$\frac{q^3}{\nabla T} = K_{\mathrm{GM}}^3 f(T_\lambda) \cdot g(T) \tag{8}$$

The $f(T\lambda)$ function decreases monotonically with the decrease of pressure. The $g(T)$ function is shown in Fig. 2, it approaches zero at low temperatures and at the Lambda points. There is again a strong resemblance of the function throughout the pressure range selected. The function $g(T)$ is proportional to the apparent thermal conductivity of the He II inside the duct. The behavior predicted by Eq. (5) agrees with data [8].

Fig. 3 presents the results from saturated and pressurized He II [3,8–12]. Eq. (4) may be re-written as [3]

$$N_q\left(\frac{\rho}{\rho_s}\right) = K_{\mathrm{GM}}\left[N_{\nabla T}\left(\frac{\rho_s}{\rho_n}\right)\right]^{1/3} \tag{9}$$

It is seen that the data in Fig. 3 are correlated satisfactorily by Eq. (9) within data scatter and property uncertainty. The $K_{\mathrm{GM}}$ data appear to decrease as $P$ is raised. Fig. 4 shows the GM-constant $K_{\mathrm{GM}} = K_{\mathrm{GM}}(P)$ for the data set of Refs. [8,10,11]. However, the departure from the $K_{\mathrm{GM}}$ value of 11.3 for saturated liquid He II to pressurized liquid He II, up to around 7 bar, is about 10%. The scarcity and the heat flux density data at higher pressures and the uncertainty of the values of the shear viscosity makes the deviation of the $K_{\mathrm{GM}}$ as much as 20% from the constant of 11.3 at the highest pressure of 20 bar.

## 4. Transport at large $\Delta T$-values

The heat flux density can reach its limiting values for larger temperature differences. The thermo physical properties can vary substantially along the length of the GM-duct. Therefore, the GM-equation must be integrated from the bath temperature to the lambda temperature at the pressure of the experiment. The experimental value of $\Delta T$ was used in the integrated GM-equation for a duct of length $L$. For the ease of integration, reduced properties are integrated against the reduced temperature $t = T/T_\lambda$. This equation may be written as

$$q_L L^{1/3} = K_{\mathrm{GM}}(\rho_\lambda^2 S_\lambda^4 T_\lambda^4 \eta_\lambda)^{1/3} \times \left\{\int_{t_b}^{1}\left(1 - \frac{\rho_n}{\rho}\right)^4 \left(\frac{\rho_n}{\rho}\right)^{-1} \left(\frac{\rho}{\rho_\lambda}\right)^2 \left(\frac{S}{S_\lambda}\right)^4 \left(\frac{\eta}{\eta_\lambda}\right) t^3\,\mathrm{d}t\right\}^{1/3} \tag{10}$$

$$y(T_\lambda) = (\rho_\lambda^2 S_\lambda^4 T_\lambda^4 \eta_\lambda)^{1/3} \tag{11}$$

$$z(t_b) = \left\{\int_{t_b}^{1}\left(1 - \frac{\rho_n}{\rho}\right)^4 \left(\frac{\rho_n}{\rho}\right)^{-1} \left(\frac{\rho}{\rho_\lambda}\right)^2 \left(\frac{S}{S_\lambda}\right)^4 \left(\frac{\eta}{\eta_\lambda}\right) t^3\,\mathrm{d}t\right\}^{1/3} \tag{12}$$

Eq. (10) reduces to

$$q_L L^{1/3} = K_{\mathrm{GM}}\, y(T_\lambda) z(t_b) \tag{13}$$

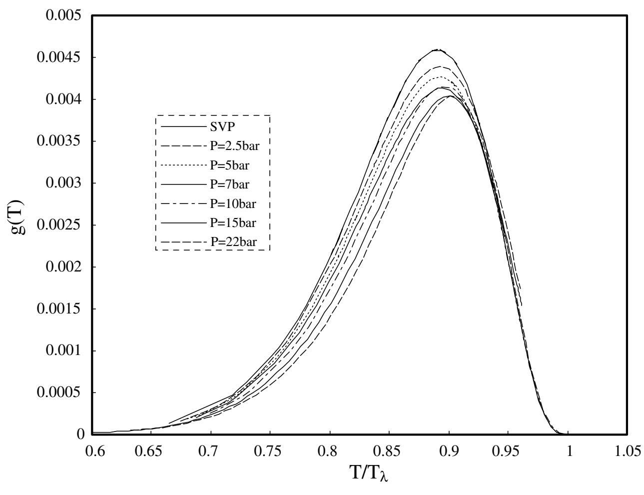


Fig. 2. The $g(T)$ function versus the reduced temperature at different pressures.

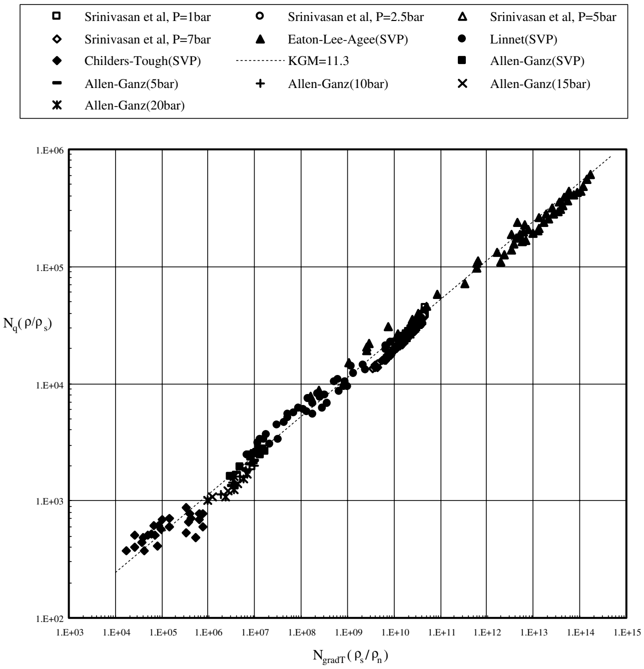


Fig. 3. The dimensionless heat flux density versus the dimensionless generalized driving force.

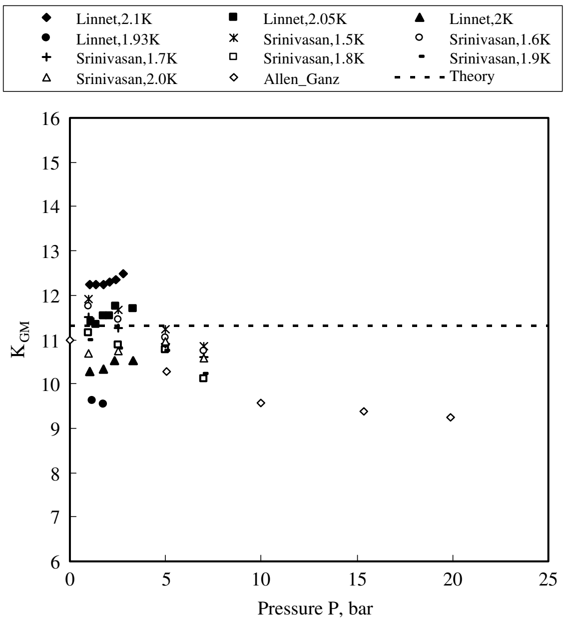


Fig. 4. The G–M constant $K_{GM}$ as a function of pressure. The dash line is for $K_{GM} = 11.3$.

The function $y(T_\lambda)$ does not vary too much with changing pressure. The function $z(t_b)$, which is a dimensionless, decreases gradually and monotonically at lower temperatures. However, it is strongly dependent on the temperature when the reduced bath temperature is increased above around 0.8. However, it does not appear to vary much with the increase of pressure. As shown in Fig. 5, the limiting heat flux density calculated by Eq. (13) is in good agreement with experimental data obtained by several authors [10,11,13–15].

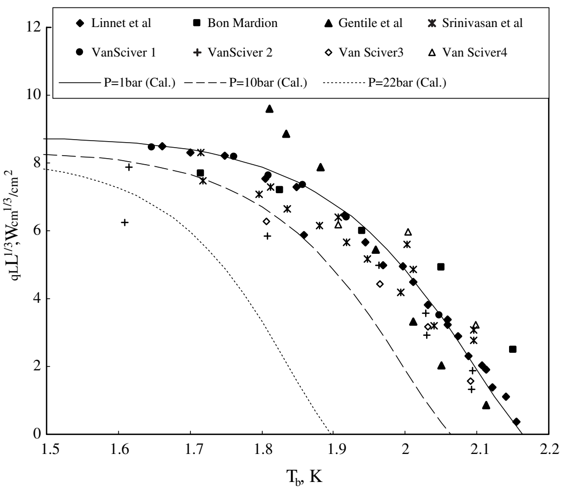


Fig. 5. Limiting heat flux density versus He II bath temperature. Experimental data are all taken at $P = 1$ bar.

## 5. Conclusion

A simplified model which relates thermo physical properties of pressurized He II is developed. The relationship between the heat flux density and the temperature gradient in a G-M tube under a given pressure can be determined by the reduced thermo physical properties with respect to their values at the Lambda point. Based on this model, the limiting heat flux density can be predicted with the ease of integration of thermal physical properties along the GM tubes. The integration values are also presented graphically so that they can be used for the first order engineering design of superconducting magnets in pressurized He II. At higher pressures, larger than 7 bar, the scarcity of data for the shear viscosity $\eta_n$ creates the uncertainty in $K_{GM}$. Other information related the fine adjustment of the power exponent in the dimensionless representation can be found in another reference [16].

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